Revised Mean Absolute Percentage Errors (MAPE) on Errors from Simple Exponential Smoothing Methods for Independent Normal Time Series

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ABSTRACT

Mean Absolute Deviation (MAD) and author's revised Mean Absolute Percentage Errors (RMAPE) are applied to measure the forecasting accuracy from different Simple Exponential Smoothing (SES) Methods for independent normal time series. When initial forecast values are derived from a seven period simple moving average, simulation results show that for independent normal time series with moderately large coefficients of variations (c.v.), such as when c.v. is greater than 0.2, the complexity from the ratios of RMAPE will mislead researchers on distinguishing the forecasting accuracies from different SES Methods. Thus, when data are from independent normal time series, the RMAPE does not reveal valid forecasting accuracies from various SES Methods, and is not recommended as an alternative for MAD.

INTRODUCTION

In simple moving average methods with a moving period of k, past k period observations are treated equally and all preceding observations are disregarded. Naturally, past data should be discounted in a more gradual way. That is, the most recent observation should get a little more weight than the 2nd most recent observation, and the 2nd most recent observation should get a little more weight than the 3rd most recent observation, and so on. Brown (1959, 1963) in the late 1950s proposed the simple exponential smoothing (SES) method which accomplishes this, and became widely used in both industry and business forecasts (Dalrymple, 1987; Edwards & Holt, 2001; Gardner, 1985; Makridakis, Wheelwright & Hyndman, 1997; Sanders & Ritzman, 1989; Winklhofer, Diamantopoulos & Witt, 1996).

Let A_t and F_t denote the actual and forecasting values at time t, respectively. Brown defines the forecast at time t+1, F_{t+1}, to be the adjustment of last period's forecast, F_t, by a fraction (α) from last period's forecasting error, $A_t - F_t$, where α is a number between 0 and 1 called the smoothing constant. In other words, we have

$$F_{t+1} = F_t + \alpha(A_t - F_t) \qquad (1)$$

By combining like terms, we also can obtain the equivalent of the above formula to be $F_{t+1} = \alpha A_t + (1-\alpha)F_t$. In order to find F_{t+1}, an initial forecast for F_t at time t is required. In this paper, we use the commonly used native forecast, A_{t-1}, as the initial forecast F_t at time t.

From the above recursive equation, the following is the reason people named it the exponentially smoothed forecast.

$$F_{t+1} = \alpha A_t + \alpha(1-\alpha)A_{t-1} + \alpha(1-\alpha)^2A_{t-2} + \alpha(1-\alpha)^3A_{t-3} + \alpha(1-\alpha)^5A_{t-4} + \ldots.. \qquad (2)$$

Brown (1959, 1963) also pointed out that for independent time series, $\alpha = 0.1$, 0.2, and 0.3 are adequate for forecasting purposes. It is also not difficult to prove that for independent time series, the variance for forecasting errors at time t+1, $A_{t+1} - F_{t+1}$, is $[1+\frac{\alpha}{2-\alpha}]\sigma^2$, where σ^2 is the variance for the independent time series, A_t. Therefore, when $\alpha = 0.1$, 0.2, and 0.3, the corresponding variance for forecasting errors at time t+1, $A_{t+1} - F_{t+1}$, will be $0.0526\,\sigma^2$, $0.1111\,\sigma^2$, and $0.2121\sigma^2$, respectively. That is, for an independent normal time series, SES with $\alpha = 0.1$ should perform better than SES with $\alpha = 0.2$ and 0.3 when Mean Absolute Deviations (MAD) and the Mean Squared Error (MSE) are used to evaluate the forecasting errors. Consequently, SES methods with $\alpha = 0.1$, 0.2, and 0.3 should perform better than SES methods with $\alpha = 0.7$, 0.8, and 0.9 when Mean Absolute Deviations (MAD) and the Mean Squared Error (MSE) are used to evaluate the forecasting errors.

Besides MAD and MSE, Mean Absolute Percentage Errors (MAPE), $(\sum_{i=1}^{k}(\frac{|A_t - F_t|}{A_t})/k$, is another widely used accuracy measurement in forecasting with non-negative actual observations, for instance, on monthly or quarterly sales, tourism forecasts, economic indicators, etc. (Chen, Bloomfield & Fu, 2003; Song, Witt & Jensen, 2003; Swanson, Tayman & Barr, 2000; Wang & Liu, 2005; Weller, 1989; Weller, 1989). However, in practical forecasts such as forecasts on profits, actual observations may end up with negative values. In this paper, a revised definition for Mean Absolute Percentage Error (RMAPE), $(\sum_{i=1}^{k}(|\frac{A_t - F_t}{A_t}|)/k$, is considered (Ren, 2007).

Many researchers (Chatfield, 1988) believe that MAD and MSE are not appropriate forecasting accuracy measurements because a few large observations can dominate the measurement. Since the MAPE expresses the forecasting errors from different measurement units into percentage errors on actual observations, it is unit free; therefore, the MAPE is probably the most widely used forecasting accuracy measurement of this kind (Goodwin & Lawton, 1999). One common criticism of the MAPE is on its existence when the actual observation A_t is equal to 0. Makridakis (1993) also argued that the MAPE is asymmetric in that "equal errors above the actual value result in a greater Absolute Percentage Error than those below the actual value". Similarly, Armstrong and Collopy (1992) stated that "the MAPE … put a heavier penalty on forecasts that exceed the actual than those that are less than the actual."

Ren (2007) pointed out that neither MAPE nor RMAPE is a sensitive forecasting accuracy measurement for comparing different Simple Average Methods with moving periods (p) of 1, 3, 5, 7, 9, and averaging periods (k) of 3, 5, 7, and 9, on independent normal time series with coefficients of variation between 0.2 to 2.0. In this paper, we find out that the complexity from MAPE and RMAPE also applies to those popular independent normal time series when SES methods are applied with an initial forecast value derived from a seven period simple moving average, and the c.v. of the independent normal time series is greater than 0.2.

DATA ANALYSIS

32,000 random data are simulated from each of Normal Distributions with a mean of 1 and a standard deviation of 0.1, 0.2, 0.3, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0 and 5.0 (i.e., with coefficients of variation (c.v.) of 0.1, 0.2, 0.3, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0 and 5.0). Data is then grouped into 1,000 groups with 32 observations each. The first seven observations are historical observations; a simple average of these seven observations is used to derive our initial forecast value. Conventionally, either a naïve or three period moving average of historical values is used to derive initial forecast values; a moving average period of seven is implemented in this case to derive a more conservative estimate (Stevenson, 2008). Simple Exponential Smoothing methods with simple exponential smoothing coefficient α of 0.1, 0.3, 0.7, and 0.9 are applied by using the formula $F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1}) = \alpha A_{t-1} + (1- \alpha)F_{t-1}$, where t=8, 9, …, 32.

Absolute Deviation $|A_t-F_t|$ and Absolute Percentage Error, $|\frac{A_t - F_t}{A_t}|$, are calculated for t=8 to 32. For independent normal time series, MSE and MAD will generate the same result. As such, Mean Absolute Deviation (MAD) and Revised Mean Percentage Error (RMAPE) of length k = 25 is studied.

In addition, 17,000 random data are simulated from each of Normal Distributions with a mean of 1 and a standard deviation of 0.1, 0.2, 0.3, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0 and 5.0 (i.e., with coefficients of variation (c.v.) of 0.1, 0.2, 0.3, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0 and 5.0). Data is then grouped into 1,000 groups with 32 observations each. The first seven observations are historical observations; a simple average of these seven observations is used to derive our initial forecast value. Simple Exponential Smoothing methods with simple exponential smoothing coefficient α of 0.1, 0.3, 0.7, and 0.9 are applied by using the formula $F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1}) = \alpha A_{t-1} + (1- \alpha)F_{t-1}$, where t=8, 9, …, 17.

Absolute Deviation $|A_t-F_t|$ and Absolute Percentage Error, $|\frac{A_t - F_t}{A_t}|$, are calculated for t=8 to 17. For independent normal time series, MSE and MAD will generate the same result. In this way, the Mean Absolute Deviation (MAD) and Revised Mean Percentage Error (RMAPE) of length k = 10 is studied.

A numerical example for simulated data from a normal distribution with a c.v. of 1 is listed in Tables 1-3. For instance, the bolded and italicized figure ***1.4713*** is the initial forecast from the seven period simple moving average. The bolded and italicized figure ***1.4265*** is derived from the Simple Exponential Smoothing method with α = 0.1 for period 9, which by formula is 1.4713+(0.1)(1.0234-1.4713)=(0.1)(1.0234)+(1-0.1)(1.4713)=1.4265. We can obtain the remaining figures similarly.

The bolded and italicized figures ***0.9596*** and ***0.4021*** in Table 2 and 3 are the absolute deviation (AD) and absolute percentage error (APE), respectively, at period 9 from the Simple Exponential Smoothing method with α = 0.1 from Table 1. They computed as |2.3861-1.4265 | =0.9596 and |2.3861-1.4265|/|2.3861|=0.4021, respectively.

The Mean Absolute Deviations (MAD) and the Revised Mean Absolute Percentage Errors (RMAPE) for the length of k=10 in Tables 2 and 3 are obtained by finding the average of absolute deviations and mean absolute percentage errors for periods 8-17.

Time	Actual	Forecast F_t			
t	A_t	EXP(0.1)	EXP(0.3)	EXP(0.7)	EXP(0.9)
1	2.1614				
2	1.5893				
3	1.9717				
4	1.2717				
5	0.5035				
6	1.2334				
7	1.5684				
8	1.0234	***1.4713***	1.4713	1.4713	1.4713
9	2.3861	***1.4265***	1.3370	1.1578	1.0682
10	3.1911	1.5225	1.6517	2.0176	2.2543
11	0.7809	1.6894	2.1135	2.8391	3.0974
12	0.1676	1.5985	1.7137	1.3983	1.0125
13	0.1675	1.4554	1.2499	0.5368	0.2521
14	2.0299	1.3266	0.9252	0.2783	0.1759
15	0.2438	1.3970	1.2566	1.5044	1.8445
16	2.2785	1.2816	0.9528	0.6220	0.4039
17	1.6060	1.3813	1.3505	1.7816	2.0911

Table 1: Forecasts from SES with α= 0.1, 0.3, 0.7, and 0.9 with c.v.=1

Time	Actual	Absolute Deviation			
t	A_t	EXP(0.1)	EXP(0.3)	EXP(0.7)	EXP(0.9)
1	2.1614				
2	1.5893				
3	1.9717				
4	1.2717				
5	0.5035				
6	1.2334				
7	1.5684				
8	1.0234	0.4479	0.4479	0.4479	0.4479
9	2.3861	***0.9596***	1.0491	1.2283	1.3179
10	3.1911	1.6686	1.5394	1.1735	0.9368
11	0.7809	0.9085	1.3326	2.0582	2.3165
12	0.1676	1.4309	1.5462	1.2308	0.8450
13	0.1675	1.2880	1.0824	0.3693	0.0846
14	2.0299	0.7033	1.1048	1.7517	1.8540
15	0.2438	1.1531	1.0128	1.2606	1.6007
16	2.2785	0.9969	1.3258	1.6566	1.8747
17	1.6060	0.2247	0.2555	0.1756	0.4851
MAD		0.9781	1.0697	1.1352	1.1763

Table 2: Absolute Deviations and MAD from SES with α= 0.1, 0.3, 0.7, and 0.9 with c.v.=1

Time	Actual	Absolute Percentage Error			
t	A_t	EXP(0.1)	EXP(0.3)	EXP(0.7)	EXP(0.9)
1	2.1614				
2	1.5893				
3	1.9717				
4	1.2717				
5	0.5035				
6	1.2334				
7	1.5684				
8	1.0234	0.4376	0.4376	0.4376	0.4376
9	2.3861	***0.4021***	0.4397	0.5148	0.5523
10	3.1911	0.5229	0.4824	0.3677	0.2936
11	0.7809	1.1634	1.7066	2.6357	2.9666
12	0.1676	8.5396	9.2273	7.3450	5.0427
13	0.1675	7.6913	6.4639	2.2056	0.5052
14	2.0299	0.3465	0.5442	0.8629	0.9133
15	0.2438	4.7296	4.1539	5.1704	6.5653
16	2.2785	0.4375	0.5819	0.7270	0.8227
17	1.6060	0.1399	0.1591	0.1093	0.3020
RMAPE		2.4410	2.4196	2.0376	1.8401

Table 3: Absolute Percentage Errors (APE) and RMAPE from SES with α= 0.1, 0.3, 0.7, and 0.9 with c.v.=1

We apply the above process to 1,000 sets of simulated time series for coefficients of variation, c.v. = 0.1, 0.2, 0.3, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0 and 5.0 for both k=10 and k=25. The time series consist of 17and 32 observations, respectively, where the first 7 observations of each series are the historical observations used to derive initial forecast value for period 8. Pairwise T-tests on each of the 1,000 independent MAD's and RMAPE's for a given c.v. are used to test whether there is difference between the means of MAD's and RMAPE's from forecasting methods Exp(0.1), EXP(0.3), EXP(0.7), and EXP(0.9), where EXP(α) represents the SES Method with a coefficient of α. Means and standard deviations of the 1,000 independent MAD's and RMAPE's are listed in Tables 4 and 5 for k=10 and k=25, respectively. P-values for the pairwise T-tests for each c.v. corresponding to k=25 are listed in Table 6. Similar results can be obtained for each c.v. corresponding to k = 10.

			MAD				RMAPE			
			Exp(0.1)	Exp(0.3)	Exp(0.7)	Exp(0.9)	Exp(0.1)	Exp(0.3)	Exp(0.7)	Exp(0.9)
k=10	c.v.=0.1	mean	0.0819	0.0861	0.0977	0.1056	0.0864	0.0901	0.1016	0.1095
		s.d.	0.0209	0.0226	0.0274	0.0308	0.0689	0.0685	0.0703	0.0724
	c.v.=0.2	mean	0.1629	0.1713	0.1950	0.2110	0.1793	0.1869	0.2106	0.2269
		s.d.	0.0384	0.0414	0.0503	0.0568	0.0559	0.0572	0.0662	0.0731
	c.v.=0.3	mean	0.2467	0.2593	0.2933	0.3173	0.3504	0.3602	0.3932	0.4177
		s.d.	0.0608	0.0652	0.0788	0.0893	0.3542	0.3488	0.3675	0.3844
	c.v.=0.5	mean	0.4069	0.4275	0.4869	0.5275	2.2882	2.2086	2.0274	1.9609
		s.d.	0.0983	0.1075	0.1326	0.1521	22.2265	18.8363	12.3070	9.5503
	c.v.=1.0	mean	0.8435	0.8629	0.9719	1.0500	5.9571	5.1482	4.9309	5.2580
		s.d.	0.2002	0.2146	0.2579	0.2917	25.7094	20.3605	17.0818	17.8537
	c.v.=1.5	mean	1.2385	1.2832	1.4464	1.5590	3.2446	4.1674	5.4253	5.9522
		s.d.	0.2956	0.3242	0.3934	0.4442	10.1746	13.1028	18.4054	20.8631
	c.v.=2.0	mean	1.6322	1.7189	1.9515	2.1076	6.8511	5.2912	6.3313	7.1026
		s.d.	0.3883	0.4208	0.5157	0.5831	76.9857	40.3007	48.9151	56.8851
	c.v.=2.5	mean	2.1032	2.1857	2.4679	2.6638	5.8320	10.1376	14.4859	16.3670
		s.d.	0.4841	0.5259	0.6412	0.7284	120.3637	223.6363	302.5191	336.5388
	c.v.=3.0	mean	2.4233	2.5552	2.9059	3.1455	3.7379	4.1304	4.7160	5.3453
		s.d.	0.5767	0.6308	0.7641	0.8679	42.8268	42.3947	24.2152	23.7347
	c.v.=4.0	mean	3.2948	3.4271	3.8817	4.1963	3.7567	3.6164	4.4856	5.1478
		s.d.	0.7871	0.8595	1.0348	1.1720	26.0747	22.8402	17.9085	17.2827
	c.v.=5.0	mean	4.0598	4.2651	4.8371	5.2362	3.2154	4.9806	8.5459	10.7375
		s.d.	1.0185	1.1096	1.3446	1.5238	21.2700	46.7798	106.6054	139.2268

Table 4: Means and Standard Deviations for MAD and RMAPE for k=10

			MAD				RMAPE			
			Exp(0.1)	Exp(0.3)	Exp(0.7)	Exp(0.9)	Exp(0.1)	Exp(0.3)	Exp(0.7)	Exp(0.9)
k=25	c.v.=0.1	mean	0.0828	0.0867	0.0983	0.1064	0.0774	0.0893	0.1009	0.1090
		s.d.	0.0131	0.0142	0.0172	0.0194	0.0123	0.0302	0.0315	0.0327
	c.v.=0.2	mean	0.1635	0.1721	0.1961	0.2126	0.1752	0.1861	0.2111	0.2279
		s.d.	0.0257	0.0283	0.0348	0.0395	0.0335	0.0364	0.0432	0.0482
	c.v.=0.3	mean	0.2613	0.2654	0.2990	0.3236	0.3965	0.3826	0.4120	0.4371
		s.d.	0.0377	0.0414	0.0508	0.0577	0.3111	0.3057	0.3162	0.3287
	c.v.=0.5	mean	0.4109	0.4317	0.4909	0.5328	2.6000	2.7814	2.9751	3.0581
		s.d.	0.0622	0.0673	0.0810	0.0930	14.8333	16.8232	19.8850	20.8837
	c.v.=1.0	mean	0.8169	0.8630	0.9825	1.0656	5.1927	5.2218	5.6830	6.0310
		s.d.	0.1213	0.1315	0.1605	0.1817	18.5756	18.9203	25.1333	27.8017
	c.v.=1.5	mean	1.2422	1.3054	1.4883	1.6119	5.2354	4.9345	4.8738	5.5002
		s.d.	0.1966	0.2119	0.2562	0.2878	25.7060	24.2341	11.0343	14.5799
	c.v.=2.0	mean	1.6482	1.7306	1.9713	2.1387	5.3418	5.6606	4.7781	7.0352
		s.d.	0.2536	0.2756	0.3330	0.3778	74.2411	62.3818	11.2320	54.0367
	c.v.=2.5	mean	2.0416	2.1484	2.4476	2.6516	6.2774	7.6095	11.3651	13.0016
		s.d.	0.3165	0.3474	0.4235	0.4815	76.1008	113.3136	182.1022	211.5296
	c.v.=3.0	mean	2.4874	2.6178	2.9870	3.2384	4.2911	4.0166	5.0103	6.1204
		s.d.	0.3722	0.4073	0.5071	0.5805	22.1601	13.7109	12.2351	19.1040
	c.v.=4.0	mean	3.3192	3.4715	3.9498	4.2823	3.2068	4.8383	4.9903	5.8167
		s.d.	0.5171	0.5587	0.6755	0.7638	33.3974	53.0497	21.0808	20.8386
	c.v.=5.0	mean	4.0888	4.3143	4.9246	5.3466	2.6268	4.2746	6.8919	8.3125
		s.d.	0.6095	0.6598	0.8084	0.9303	12.8948	28.1843	58.0033	73.2569

Table 5: Means and Standard Deviations for MAD and RMAPE for k=25

K=25		MAD				RMAPE			
		Exp(0.1)	Exp(0.3)	Exp(0.7)	Exp(0.9)	Exp(0.1)	Exp(0.3)	Exp(0.7)	Exp(0.9)
c.v.=0.1	Exp(0.1)	---	0.0000	0.0000	0.0000	---	0.0000	0.0000	0.0000
	Exp(0.3)		---	0.0000	0.0000		---	0.0000	0.0000
	Exp(0.7)			---	0.0000			---	0.0000
c.v.=0.2	Exp(0.1)	---	0.0000	0.0000	0.0000	---	0.0000	0.0000	0.0000
	Exp(0.3)		---	0.0000	0.0000		---	0.0000	0.0000
	Exp(0.7)			---	0.0000			---	0.0000
c.v.=0.3	Exp(0.1)	---	0.0000	0.0000	0.0000	---	0.0000	0.0000	0.0000
	Exp(0.3)		---	0.0000	0.0000		---	0.0000	0.0000
	Exp(0.7)			---	0.0000			---	0.0000
c.v.=0.5	Exp(0.1)	---	0.0000	0.0000	0.0000	---	0.0416	**0.1095**	**0.1060**
	Exp(0.3)		---	0.0000	0.0000		---	**0.1916**	**0.1646**
	Exp(0.7)			---	0.0000			---	**0.1125**
c.v.=1.0	Exp(0.1)	---	0.0000	0.0000	0.0000	---	**0.7797**	**0.1513**	**0.0534**
	Exp(0.3)		---	0.0000	0.0000		---	**0.1030**	0.0360
	Exp(0.7)			---	0.0000			---	0.0017
c.v.=1.5	Exp(0.1)	---	0.0000	0.0000	0.0000	---	0.0037	**0.5696**	**0.6232**
	Exp(0.3)		---	0.0000	0.0000		---	**0.9205**	**0.2814**
	Exp(0.7)			---	0.0000			---	0.0002
c.v.=2.0	Exp(0.1)	---	0.0000	0.0000	0.0000	---	**0.4215**	**0.8071**	0.0200
	Exp(0.3)		---	0.0000	0.0000		---	**0.6462**	0.0002
	Exp(0.7)			---	0.0000			---	**0.1658**
c.v.=2.5	Exp(0.1)	---	0.0000	0.0000	0.0000	---	**0.2713**	**0.1309**	**0.1179**
	Exp(0.3)		---	0.0000	0.0000		---	**0.0857**	**0.0836**
	Exp(0.7)			---	0.0000			---	**0.0799**
c.v.=3.0	Exp(0.1)	---	0.0000	0.0000	0.0000	---	**0.3743**	**0.1682**	0.0000
	Exp(0.3)		---	0.0000	0.0000		---	0.0000	0.0000
	Exp(0.7)			---	0.0000			---	0.0000
c.v.=4.0	Exp(0.1)	---	0.0000	0.0000	0.0000	---	0.0093	0.0006	0.0001
	Exp(0.3)		---	0.0000	0.0000		---	**0.8890**	**0.4239**
	Exp(0.7)			---	0.0000			---	0.0000
c.v.=5.0	Exp(0.1)	---	0.0000	0.0000	0.0000	---	0.0026	0.0051	0.0045
	Exp(0.3)		---	0.0000	0.0000		---	0.0079	0.0060
	Exp(0.7)			---	0.0000			---	0.0051

Table 6: p-values from Pairwise T-tests for k=25

CONCLUSION

This study demonstrates that for independent normal time series with c.v. between 0.1 to 5.0, the MAD can clearly show the differences from different SES methods of EXP(0.1), EXP(0.3), EXP(0.7), and EXP(0.9). From Tables 6 and 7, we can see that MAD generated by EXP(0.1), EXP(0.3), EXP(0.7), and EXP(0.9) are in ascending order for independent normal time series. In other words, EXP(0.1) or EXP(0.3) will be preferred for those independent normal time series, which is consistent to what Brown (1959, 1963) claimed.

In contrast, the RMAPE sometimes does little to differentiate between the different SES methods when c.v. is moderately large; that is, when c.v. is greater than 0.2. The complexity from the ratios will be released only when the c.v. is very small, say less than 0.2, or when the c.v. is very large, say greater than 4, in addition to k being large. Therefore, from our study, the RMAPE should only be used to measure the forecasting accuracy for independent normal time series when the series' c.v. is very large (e.g., much greater than 4.0) and k is large, or when the series' c.v. is very small (e.g. less than 0.2) when SES methods are applied. In general, the MAD is recommended for all independent normal time series.

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