

# **Optimizing Subsidies for the Location of Distribution Centers**

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## **ABSTRACT**

Location problems have been discussed for many decades. Hakimi (1964) was the first to introduce location models to the operations research community, and he was also the first to put location problems into the context of networks. Since then, thousands of papers have been written on various aspects of location planning. Virtually all of these papers include two independent economic agents. First, there are the customers, who have a certain (fixed or variable) demand that they attempt to satisfy. If given the choice, they will purchase from the least expensive source. Secondly, the firms will choose their locations, so as to either maximize their profits or minimize their costs. Which objective they choose depends on their pricing policy: in case of mill pricing, customers pick up the goods themselves and pay for their own transportation, so that this policy reduces to the maximization of “capture,” i.e., the sales and associated revenue that result from the locational arrangement. On the other hand, in case of delivered pricing (often in the form of the related policy of uniform delivered pricing), firms pay for the cost of transportation and, given that the demand does not change with the location of the firm, the model reduces to cost minimization.

The focus of the paper presented here is different. First, there are three, rather than two, levels of decision makers. On the lowest level there are the customers, on the next-higher level there is again the firm’s decision maker, while on the highest level there is a regional planner. The customers purchase a known quantity of a good from the firm, the firm will decide how many distribution centers it will locate and where they should be sited, while the regional planner will

offer subsidies to the firm to locate at points that are deemed important by the planner. Each economic agent has his own objective function: consumers are only interested in satisfying their demand, the firm will attempt to maximize its profit, and the regional planner will maximize its own revenue minus the subsidy that is required to obtain it. It is worth noting that while many regional planners use non-economic reasons for favoring certain location decisions (e.g., social policy), we assume that the regional planner's motives are purely economic (and, as such, quantifiable).

Ever since the 1930s, subsidies have played an ever-increasing role in managerial decision making in general and location planning in particular. Bennett (1980) provides an excellent case study of a firm and the use of its market power. While the proliferation of subsidies in practice is remarkable, subsidy models such as those by Conrad (2005) typically use countries or regions that have been reduced to a point. Very few studies such as that by Eiselt (2000) use networks as the space in which location decisions are made. The main reason for this study is the quest for a model that makes reasonably realistic assumptions about the regional planner's behavior and provides the firm's location planner with a guideline to choose the location within a region.

The structure of the subsidy–location problem is one of a bilevel programming problem. Problems of this nature have first been discussed by the economist von Stackelberg (1943). The main idea is that one firm (or a group of firms) acts as the market leader, while another firm (or group of firms) acts as followers. While this two-level arrangement appears simplistic, it is frequently observed in practice. Even if multiple firms attempt to enter the market, they will not do so in the form of a waiting line as suggested by Prescott and Visscher (1977): the last entrant in their model with three firms has a significant lower profit than the first two entrants, so why would any firm accept this? On the other hand, the difference between leader and follower is clearly determined: in order to be a leader, a firm needs to have the capability to be a leader (e.g., a sufficient capital or knowledge base) and there must be a significant incentive to become a leader, something also known as the “first mover advantage” in marketing.

Followers solve a fairly easy problem: they take the decision of the market leaders as given, and optimize on that basis. The leader's problem, on the other hand, is much more complicated. In his planning, the leader will have to take the reaction of the followers into consideration. This means that the leader needs to determine the “reaction function,” i.e., it is required to determine the follower's course of action for each of the leader's possible decisions. Typically, in location problems on networks, this means that for each location decision by the leader, the optimal response by the follower must be determined by solving a mixed integer programming problem, making the leader's problem extremely hard to solve.

In our model, the position of the leader is taken by the regional planner, while the firm's decision maker is considered the follower. This is a reasonable arrangement, as the regional planner will offer subsidies by considering their effects on the firm's location and, ultimately, the regional planner's revenue. On the other hand, the firm will optimize its own profit by taking the subsidy offer as a given parameter. While in general, the leader's problem may be extremely difficult to solve, we demonstrate that in this model, the leader's problem is computationally tractable. As a

matter of fact, the computational requirements merely involve the solution of two mixed integer programming problems of moderate size.

After formulating the problem as a mixed-integer programming model, the theoretical results are then tested computationally. As far as demand data and distances are concerned we use Daskin's (2008) United States database involving a network with 150 nodes. Different scenarios are examined. In particular, we investigate the regional planner's revenue as a function of the economic multipliers that are modeling the spinoff benefits that result from the firm's capital and operational expenditures. As expected, revenue increases with the multipliers. However, when one of the multipliers approaches unity, the magnitude of the required subsidy payment drops. The reason for this counterintuitive behavior is found in the formulation. The reason would have been hidden, had the mathematical model not distinguished between multipliers for capital expenditures and those for operating expenses. It is hardly surprising that the unmet demand decreases with increasing values of the multipliers, but it was not expected that the entire demand would already be satisfied for fairly small values of the multipliers. Finally, considering different elasticities of the demand, we determine the changes in some of the key parameters, particularly the subsidy that is required to be offered to the firm by the regional planner.

A number of potential extensions exist and are discussed. An obvious generalization involves multiple regions. There are interesting case studies in the literature that focus on this case, e.g., the study of the Ford Motor Company's manufacturing plant in Europe in the early 1980s. We can demonstrate that this case is easy to solve as well. On the other hand, generalizations of the basic model to the case of even a single region but with multiple firms appears to be difficult. We close with some thoughts on variations of the model, e.g., the existence of different perceptions of the demand.

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