Capital Budgeting Decisions Using Simulation and Binary Linear Programming

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Abstract

Daxia Company case illustrates how the study of linear programming and risk analysis can be facilitated with popular spreadsheets and their simulation add-ins. From new products having a singular expected value for NPV, binary linear programming (BLP) optimally selects the combination of products that maximizes total NPV given capital constraints. However, when probability distributions are used to model risk of the products, an optimized simulation finds a different set of products for a risk adverse strategy. The purposes of the Daxia Company example are to illustrate the combining of simulations with linear programming, and to present how advances in spreadsheet technology facilitate more meaningful modeling and risk analysis of business decisions.

INTRODUCTION

Common techniques available to managers in addressing risk within a model include sensitivity analysis, scenario analysis, and recently spreadsheet simulation. @Risk and Crystal Ball are spreadsheet simulation add-ins suited for risk analysis (Kelliher *et al*., 1996; Togo, 2004). Their resulting distributions for targeted cells provide information to better manage risk for the modeled relationship. For example, in contrast to an expected value for borrowings within a cash budget, simulation generates a distribution identifying borrowings at a specified percentile of outcomes.

A constrained optimization or linear programming model addresses the problem of allocating scarce resources such that an objective function is optimized (Moore and Weatherford, 2001). Constrained linear models have two features in common: an objective function and constraints. For example, a typical cost/managerial accounting problem attempts to maximize contribution margin or minimize costs, given labor, material or sales constraints and requirements.

Spreadsheet linear programming add-ins (e.g., SOLVER and EVOLVER) also have the capabilities to perform integer and binary linear programming. Yet, integer linear programming (ILP) and binary linear programming (BLP) are seldom taught. Instead, students will round to the nearest integer even though rounding may introduce infeasibilities caused by violating constraints. Binary linear programming (BLP) models utilize integer variables (usually 1 and 0) to indicate logical or dichotomous decisions (e.g., on/off, true/false or accept/reject). Binary linear programming models are useful for scheduling, financial portfolios, capital rationing environments and production planning.

A drawback to linear programming is its inability to optimize models having probability distributions as input variables. Yet, there are spreadsheet add-ins (e.g., RiskOptimizer and OptQuest) that will optimize linear programming models having probabilistic input when a statistic is specified for an output variable. The result of this optimization with simulation is a set of values that maximizes or minimizes the objective function while meeting its desired simulation statistic and constraints.

The objective of the following example is to illustrate how simulations can be combined with linear programming. With this added capability, more managerial strategies can be evaluated such as minimizing the standard deviation of the objective function, or maximizing the objective function at the 25th percentile.

DAXIA COMPANY: A CAPITAL BUDGETING CONSTRAINT EXAMPLE

The Daxia Company example illustrates the benefits of binary linear programming, simulation, and optimization with simulation for a capital budgeting constraint problem. Requirement 1 computes the NPV for each of the four products using expected values. Requirement 2 relies on binary linear programming to obtain an optimal mix of products that maximizes the expected value for total NPV given capital constraints. Requirement 3 models uncertainty within each product, performs a simulation, and then generates a probability distribution for its NPV. Requirement 4 performs an optimization with simulation by adopting a risk adverse strategy for an optimal mix of products that maximize NPV at the 25th percentile given capital constraints. Requirement 5 compares the different outcomes for Requirements 2 and 4 using a graphical presentation for their distribution of total NPV.

The annual lease payment and the first-year net cash inflow for Daxia Company's products are listed in Panel A of Table 1. In Panel B, yearly adjustment factors to the first-year net cash inflow reflect the short five-year life cycle of the products. Panel B also lists the steadily decreasing investment capital available over the next five years. NPV computations use an 8% cost of capital and ignore income taxes.

<Insert Table 1>

Requirement 1: Table 2 is a spreadsheet that computes the NPV for each product. The PV computations assume an 8% discount rate, lease payments occurring at the beginning of the year, and annual net cash inflows available at the end of the year. The net cash inflow for a year is equal to Year 1's net cash inflow multiplied by its adjustment factor. Product A has a NPV of $117,000, which is $764,000 PV of net cash inflow less $647,000 PV of lease payment. Product B has a $204,000 NPV, Product C has a $143,000 NPV, Product D has a $120,000 NPV, and Product E has a $42,000 NPV.

<Insert Table 2>

Requirement 2: Table 3 presents the binary linear programming for this example. The present values of the yearly capital constraints are computed as additional **Input Data**. The **Setup** section uses the add-in Solver and refers to the NPV of products and the PV of lease payments of Table 2. Based on expected values, the **BLP Solution** has the optimal mix of products A, C and D maximizing total NPV = \$380,000, while meeting yearly capital constraints. Over the five-year period, these products require PV lease payments of \$2,113,000, which is less than the \$2,389,000 PV capital constraint for the same period.

<Insert Table 3>

Requirement 3: Table 1 also presents probability distributions for the first-year net cash inflow of the products. By substituting the mean value with its probability distribution, the spreadsheet model of Requirement 1 is able to perform a simulation (n=15,000) with the add-in @RISK. Figure 1 displays useful probabilistic information in a graphical format for products A and B. The symmetrical NPV graph for Product A supports a mean value of \$117,000, with 5% ≤ \$18,000 and 95% ≤ \$217,000. The triangular NPV graph for Product B has a mean of \$204,000, with 5% ≤ \$144,000 and 95% ≤ \$284,000. Output distributions provide a range of possible outcomes and their likelihood of occurrence.

<Insert Figure 1>

Requirement 4: A risk adverse solution to the capital constraint problem maximizes total NPV at the 25th percentile. Table 3 displays the **25th Percentile Solution** which uses the binary linear program of Requirement 2, the output NPV distributions for all the products from Requirement 3, and the RiskOptimizer add-in to perform an optimization with simulation. Simulation addresses the uncertainty present in the model, while optimizing algorithms generate values for the decision's adjustable cells. Hence, the decision maximizes the target cell subject to the specified statistic for its distribution and other constraints.

The mix of products from the **25th Percentile Solution** for total NPV consists of Products B, D and E. This solution has an expected mean value of \$366,000, while maximizing its 25th percentile value at \$310,000. This product mix requires \$2,026,000 of PV lease payments over the five-year period, with each year's capital constraint met.

Requirement 5: To compare the results of the **BLP Solution** and the **25th Percentile Solution,** a simulation for both was performed at the same time**.** The Total NPV graphs for both models are presented in Figure 2.

<Insert Figure 2>

At the 25th percentile, the **BLP Solution** has a NPV of \$237,000, while the **25th Percentile Solution** has a NPV of \$310,000. In comparison, the **25th Percentile Solution** is risk adverse in that its mix of products (a) generates an additional \$73,000 in NPV at the 25th percentile and (b) significantly reduces the likelihood of incurring a loss. The tradeoffs are that (a) the expected mean of \$366,000 for the **25th Percentile Solution** is \$14,000 less than the expected mean of \$380,000 for the **BLP Solution**, and (b) larger values for NPV are less likely with the **25th Percentile Solution**, which has a 95th percentile value of \$505,000 in comparison to \$729,000 for the **BLP Solution**.

SUMMARY AND CLASS EFFECTIVENESS

This example illustrates how popular spreadsheet add-ins facilitate the study of risk analysis and linear programming. The study of risk analysis was extended to spreadsheet simulation. Students learn how a simple simulation tool aids them in their quantitative analysis of uncertainty within a modeled relationship. The capital budgeting constraint example also highlighted binary linear programming for dichotomous decisions. Students are quick to see how this tool can be used in making decisions evaluating projects, investments, financial portfolios or production schedules.

And where risk can be modeled within an optimization technique, then optimization with simulation is available as another tool for analysis. By specifying another simulation statistic, such as mean, standard deviation, variance, skewness, minimum, or maximum, other managerial strategies may be examined.

Within decision sciences and other business courses, risk analysis have benefited greatly from advances in technology. Consequently, business educators are able to introduce more meaningful analysis and modeling into the classroom.

References

Kelliher, C., Fogarty, T. and Goldwater, P. (1996). Introducing uncertainty in the teaching of pensions: a simulation approach. *Journal of Accounting Education* **14**(1) Spring, 69-98.

Moore, J. H. and Weatherford, L. R. (2001). *Decision modeling with microsoft excel* (6th ed). Upper Saddle River, NJ: Prentice Hall.

Togo, D. F. (2004). Risk analysis for accounting models: a spreadsheet simulation approach. *Journal of Accounting Education* **22**(2), 153-163.

Panel A: Cash Flows (in thousands)

	Annual Lease	First-Year Net Cash Inflow Mean	First-Year Net Cash Inflow Probability Distribution
Product A	$ 150	$ 190	Normal, with $15 standard deviation
Product B	$ 270	$ 340	Triangular, with $320 minimum, $330 most likely, and $370 maximum
Product C	$ 200	$ 250	Normal, with $50 standard deviation
Product D	$ 140	$ 180	Triangular, with $160 minimum, $180 most likely, and $200 maximum
Product E	$ 60	$ 75	Normal, with $15 standard deviation
	$ 820		

Panel B: Yearly Net Cash Inflow Adjustment and Capital Constraint

	Year 1	Year 2	Year 3	Year 4	Year 5
Net Cash Inflow Adjustment Factor	1.00	1.15	1.05	.95	.85
Capital Constraint (in thousands)	$ 600	$ 575	$ 550	$ 525	$ 500

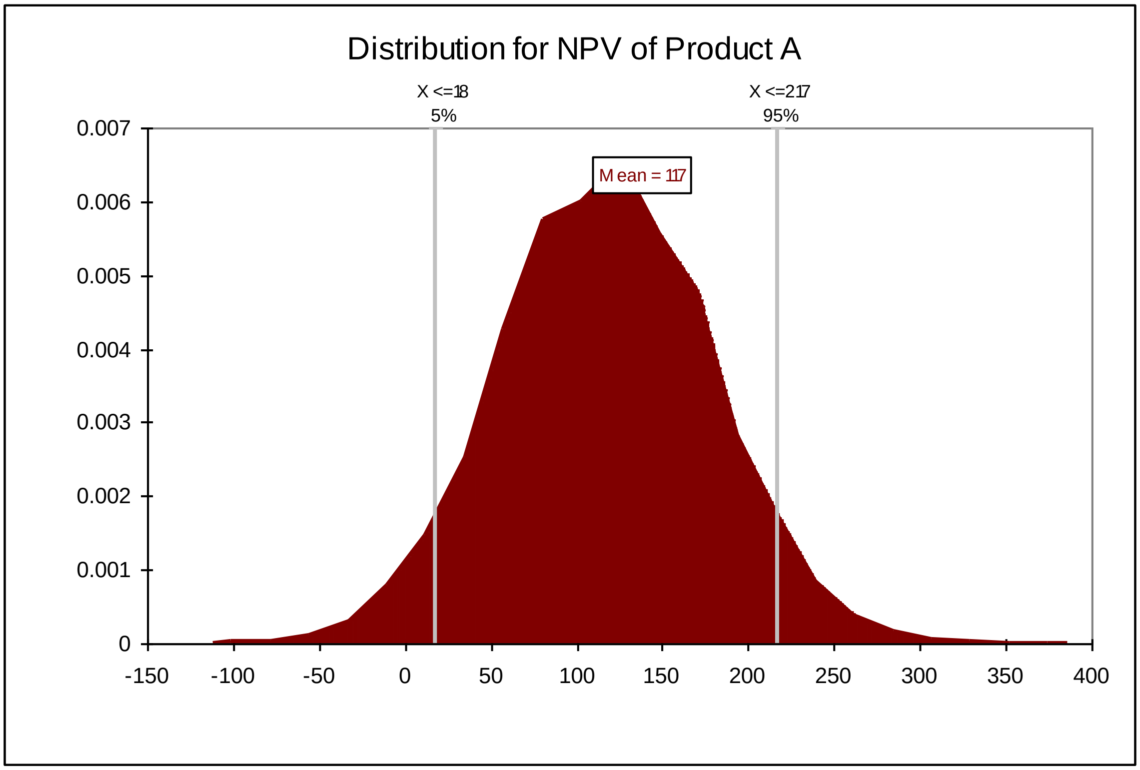
Table 1: Product Information

Input Data							
		Product A	Product B	Product C	Product D	Product E	Total
Lease payment (beginning)		150	270	200	140	60	820
First-year net cash inflow (end)		**190**	**340**	**250**	**180**	**75**	
		Year 1	Year 2	Year 3	Year 4	Year 5	
Net cash inflow adjustment factor		1.00	1.15	1.05	0.95	0.85	
NPV of Products (i=8%)	PV	Year 1	Year 2	Year 3	Year 4	Year 5	Total
Product A							
Lease payment		150	150	150	150	150	750
PV of lease payment	647	150	139	129	119	110	
Net cash inflow		190	219	200	181	162	950
PV of net cash inflow	764	176	187	158	133	110	
NPV of Product A	**117**	26	48	29	14	0	
Product B							
Lease payment		270	270	270	270	270	1350
PV of lease payment	1163	270	250	231	214	198	
Net cash inflow		340	391	357	323	289	1700
PV of net cash inflow	1367	315	335	283	237	197	
NPV of Product B	**204**	45	85	52	23	-1	
Product C							
Lease payment		200	200	200	200	200	1000
PV of lease payment	862	200	185	171	159	147	
Net cash inflow		250	288	263	238	213	1250
PV of net cash inflow	1005	231	246	208	175	145	
NPV of Product C	**143**	31	61	37	16	-2	
Product D							
Lease payment		140	140	140	140	140	700
PV of lease payment	604	140	130	120	111	103	
Net cash inflow		180	207	189	171	153	900
PV of net cash inflow	724	167	177	150	126	104	
NPV of Product D	**120**	27	47	30	15	1	
Product E							
Lease payment		60	60	60	60	60	300
PV of lease payment	259	60	56	51	48	44	
Net cash inflow		75	86	79	71	64	375
PV of net cash inflow	301	69	74	63	52	43	
NPV of Product E	**42**	9	18	12	4	-1	

Table 2: NPV of Products

Input Data	Year 1	Year 2	Year 3	Year 4	Year 5	Total		
Capital constraints	600	575	550	525	500	2750		
PV of capital constraints	600	532	472	417	368	2389		
Setup	Product A	Product B	Product C	Product D	Product E	Total		Capital
Decision	**1**	**1**	**1**	**1**	**1**	**5**		
NPV	117	204	143	120	42	**626**		
PV of lease payments								
Year 1	150	270	200	140	60	820	<=	600
Year 2	139	250	185	130	56	760	<=	532
Year 3	129	231	171	120	51	702	<=	472
Year 4	119	214	159	111	48	651	<=	417
Year 5	110	198	147	103	44	602	<=	368
Total	647	1163	862	604	259	3535		2389
BLP Solution	Product A	Product B	Product C	Product D	Product E	Total		Capital
Decision	**1**	**0**	**1**	**1**	**0**	**3**		
NPV	117	0	143	120	0	**380**		
PV of lease payments								
Year 1	150	0	200	140	0	490	<=	600
Year 2	139	0	185	130	0	454	<=	532
Year 3	129	0	171	120	0	420	<=	472
Year 4	119	0	159	111	0	389	<=	417
Year 5	110	0	147	103	0	360	<=	368
Total	647	0	862	604	0	2113		2389
25th Percentile Solution	Product A	Product B	Product C	Product D	Product E	Total		Capital
Decision	**0**	**1**	**0**	**1**	**1**	**3**		
NPV	0	204	0	120	42	**366**		
PV of lease payments						**Percentile**	**(0.25)**	**= 310**
Year 1	0	270	0	140	60	470	<=	600
Year 2	0	250	0	130	56	436	<=	532
Year 3	0	231	0	120	51	402	<=	472
Year 4	0	214	0	111	48	373	<=	417
Year 5	0	198	0	103	44	345	<=	368
Total	0	1163	0	604	259	2026		2389

Table 3: Binary Linear Programming



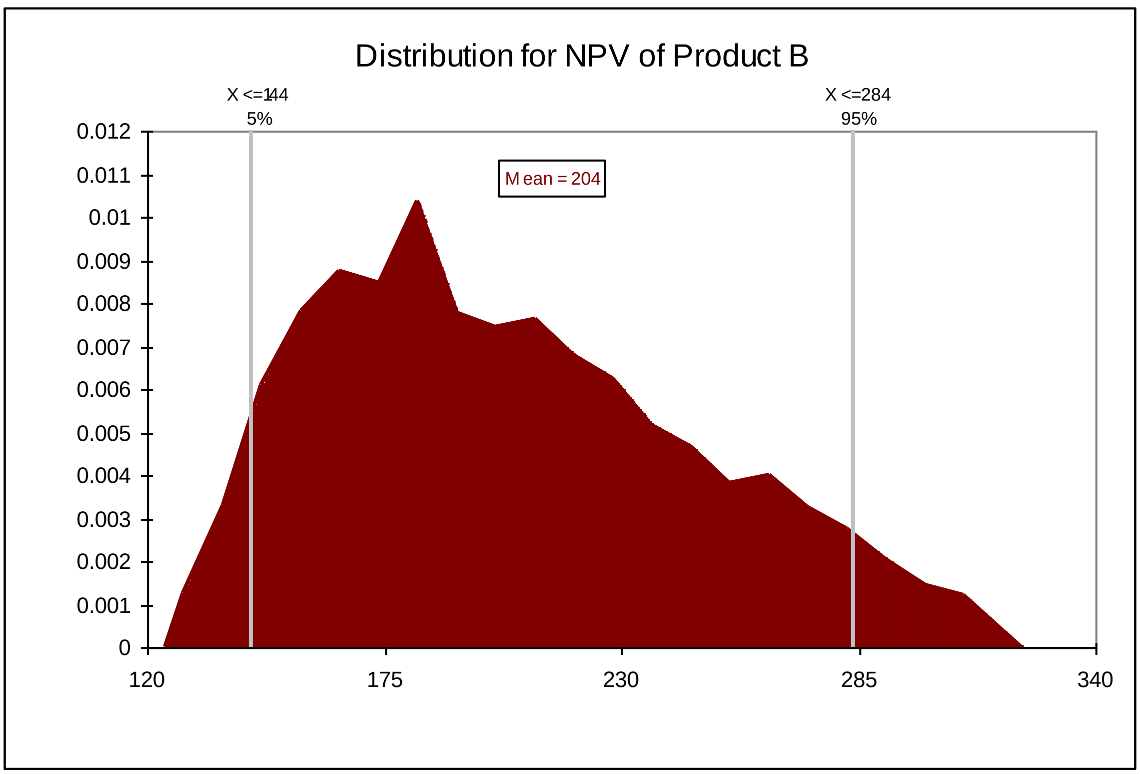
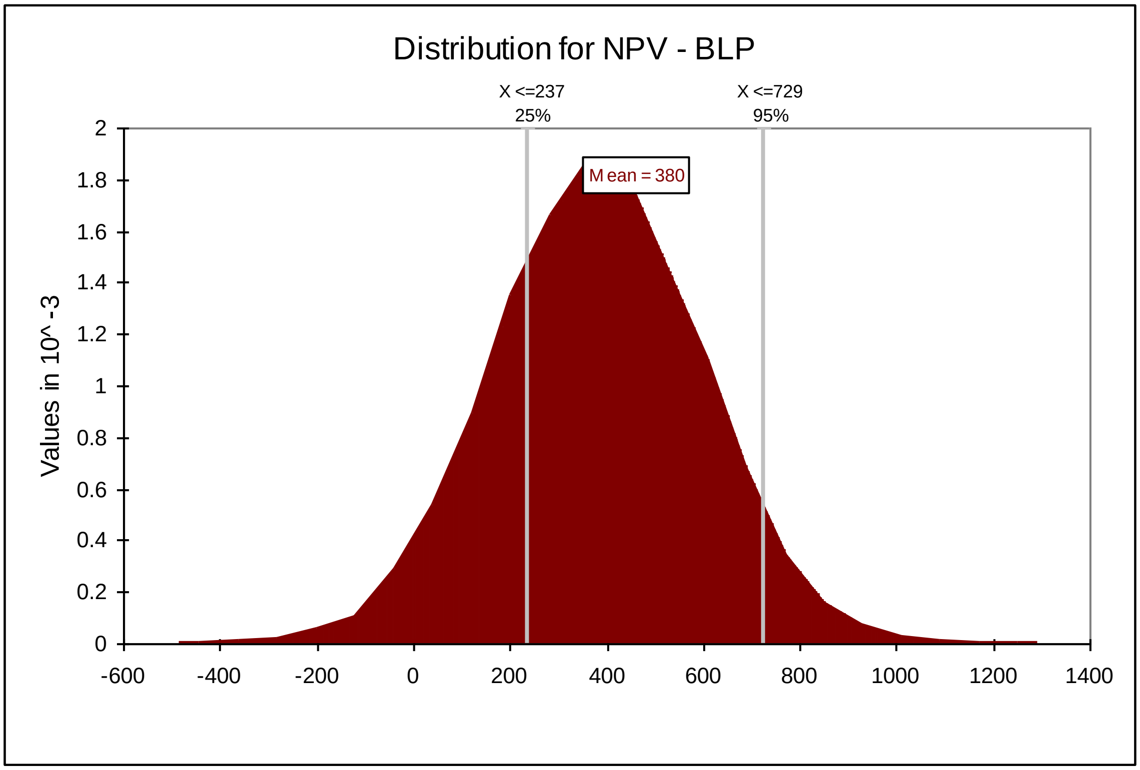


Figure 1: Product A and Product B Distributions for NPV



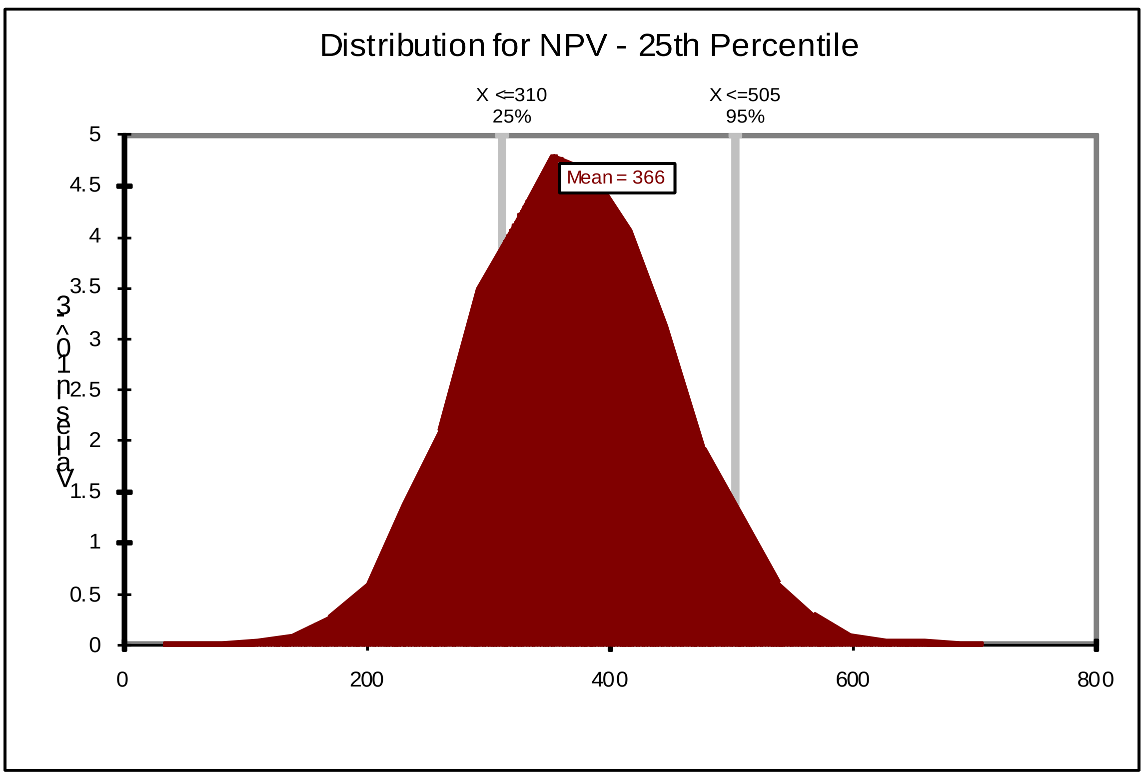


Figure 2: Total NPV - BLP and Total NPV – 25th Distributions